

FINAL TERM EXAMINATION  
Fall 2009

**Calculus & Analytical Geometry-I**

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**UPDATED VERSION**

**Exclusive thanks to Mahar Azahar (Lodhran)**

**Question No: 1 ( Marks: 1 ) - Please choose one**

$$y = \frac{x^2}{2}$$

Let  $y = \frac{x^2}{2}$ . Find average rate of change of  $y$  with respect to  $x$  over the interval  $[3, 4]$

$\frac{25}{2}$



$\frac{7}{2}$

$\frac{25}{14}$



$\frac{7}{14}$



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**Question No: 2 ( Marks: 1 ) - Please choose one**

If  $2x - y = -3$  then  $\frac{dy}{dx} =$

2

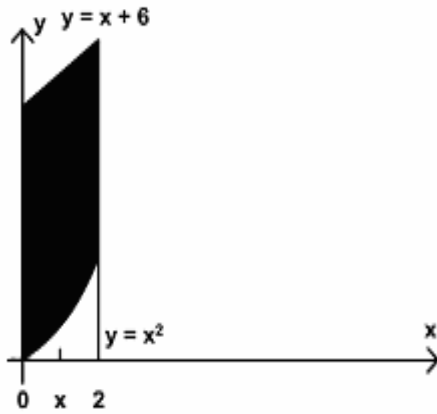
-2

0

-3

**Question No: 3 ( Marks: 1 ) - Please choose one**

In the following figure, the area bounded on the sides by the lines are :



- ▶  $x = 0$
- ▶  $x = 2$
- ▶  $x = 0$  and  $x = 2$
- ▶  $x = 6$

**Question No: 4 ( Marks: 1 ) - Please choose one**

What is the sum of following series?

$$1 + 2 + 3 + 4 + \dots + n$$

- ▶  $\frac{n+1}{2}$
- ▶  $\frac{(n+1)(n+2)}{2}$
- ▶  $\frac{n(n+2)}{2}$
- ▶  $\frac{n(n+1)}{2}$

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**Question No: 5 ( Marks: 1 ) - Please choose one**

Let  $f$  is a smooth function on  $[0, 3]$ . What will be the arc length  $L$  of the curve  $y = f(x)$  from

$x = 0$  to  $x = 3$ ?

$$L = \int_0^3 \sqrt{1 + [f'(x)]^2} dx$$

▶

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}$$



$$L = \int_0^3 \sqrt{1 + [f'(x)]^2} dy$$



$$L = \int_0^3 \sqrt{1 + [f'(x)]^2} dx$$



**Question No: 6 ( Marks: 1 ) - Please choose one**

The PYTHAGORAS theorem describes the relationship between the sides of .....▶ **Right angle triangle** .....

▶ **Right angle triangle**

▶ Isoceleous triangle

▶ Equilateral triangle

**Question No: 7 ( Marks: 1 ) - Please choose one**

Which operation can not be applied on the functions?

▶ Subtraction

▶ **Cross product**

▶ Addition

▶ Composition

**Question No: 8 ( Marks: 1 ) - Please choose one**

The graph of the equation  $y = x^2 - 4x + 5$  will represent

▶ **Parabola**

▶ Straight line

▶ Two straight lines

▶ Ellipse

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**Question No: 9 ( Marks: 1 ) - Please choose one**

Polynomials are always ..... functions

▶ **Continuous**

▶ Discontinuous

**Question No: 10 ( Marks: 1 ) - Please choose one**

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The  $\tan(x)$  is discontinuous at the points where

- ▶  $\cos(x) = 0$
- ▶  $\sin(x) = 0$
- ▶  **$\tan(x) = 0$**

**Question No: 11 ( Marks: 1 ) - Please choose one**

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A differentiable function must be differentiable on the interval

- $(-\infty, \infty)$
- ▶  $(0, \infty)$
- ▶  $(-\infty, \infty)$
- ▶  $(a, \infty)$  where  $a$  is any negative integer

**Question No: 12 ( Marks: 1 ) - Please choose one**

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Let  $y = (x^3 + 2x)^{37}$ . Which of the following is correct?

$$\frac{dy}{dx} = (37)(x^3 + 2x)^{36}$$

▶

$$\frac{dy}{dx} = 111x^2(x^3 + 2x)^{36}$$

▶

$$\frac{dy}{dx} = (111x^2 + 74)(x^3 + 2x)^{36}$$

▶

$$\frac{dy}{dx} = (111x^2 + 74)(x^3 + 2x)^{38}$$

▶

**Question No: 13 ( Marks: 1 ) - Please choose one**

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$$\int \frac{3x^2 + 4x + 1}{x^3 + 2x^2 + x - 3} dx$$

Consider the indefinite integral

Let  $t = x^3 + 2x^2 + x - 3$

Is the following substitution correct?

$$\int \frac{3x^2 + 4x + 1}{x^3 + 2x^2 + x - 3} dx = \int \frac{1}{t} dt$$

- ▶ Yes
- ▶ **No**

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**Question No: 14 ( Marks: 1 ) - Please choose one**

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$\log_b ac = \underline{\hspace{2cm}}$

▶  $\log_b a + \log_b c$

▶  $\log_b a - \log_b c$

▶  $\frac{\log_b a}{\log_b c}$

▶  $(\log_b a)(\log_b c)$



**Question No: 15 ( Marks: 1 ) - Please choose one**

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If a function has an extreme value (either a maximum or a minimum) on an open interval (a,b), then the extreme value occurs at ..... of f

▶ First point

▶ Mid point

▶ **Critical point**

▶ End point

**Question No: 16 ( Marks: 1 ) - Please choose one**

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The Mean Value Theorem states that "Let function  $f$  be differentiable on (a,b) and continuous on  $[a, b]$ , then there exist at least one point  $c$  in (a,b) where ....."

▶  $f'(c) = \frac{f(b) - f(a)}{b - a}$

▶  $f(c) = \frac{f(b) - f(a)}{b - a}$



▶  $f(c) = \frac{f(a) - f(b)}{b - a}$



▶  $f'(c) = \frac{f(a) - f(b)}{b - a}$



**Question No: 17 ( Marks: 1 ) - Please choose one**

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$$\frac{d}{dx}[F(x)] = f(x)$$

If there is some function  $F$  such that  $\frac{d}{dx}[F(x)] = f(x)$  then any function of the form  $F(x) + C$  is ..... of  $f(x)$

- ▶ Derivative
- ▶ **Antiderivative**
- ▶ Slope
- ▶ Maximum value

**Question No: 18 ( Marks: 1 ) - Please choose one**

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$$\sum_{k=1}^n f(x_k^*) \Delta x_k$$

The sum \_\_\_\_\_ is known as:

- ▶ **Riemann Sum**
- ▶ General Sum
- ▶ Integral Sum
- ▶ Geometric Sum

**Question No: 19 ( Marks: 1 ) - Please choose one**

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$$\int_0^{\frac{\pi}{2}} \cos u \, du$$

If \_\_\_\_\_, then which of the following is true?

- ▶ 2
- ▶ **1**
- ▶ 0
- ▶ -1

**Question No: 20 ( Marks: 1 ) - Please choose one**

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$$\int_0^{\pi} \sin u \, du$$

If \_\_\_\_\_, then which of the following is true?

- ▶ 1
- ▶ 2
- ▶ **0**
- ▶ -1

**Question No: 21 ( Marks: 1 ) - Please choose one**

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$$\frac{d}{dx}[F(x)] = f(x)$$

If there is some function  $F$  such that \_\_\_\_\_ then antiderivatives of  $f(x)$  are  $F(x) + C$ . What does  $C$  represents?

- ▶ Polynomial
- ▶ **Constant**
- ▶ Dependent Variable
- ▶ Independent Variable

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**Question No: 22 ( Marks: 1 ) - Please choose one**

If  $f$  and  $g$  are continuous function on an interval  $[a, b]$

and  $f(x) \geq g(x)$  for  $a \leq x \leq b$ , then area is bounded by the lines parallel to:

- ▶ X -axis
- ▶ Y-axis
- ▶ Both X -axis and Y-axis

**Question No: 23 ( Marks: 1 ) - Please choose one**

$$\int_1^{2/3} dx = \underline{\hspace{2cm}}$$

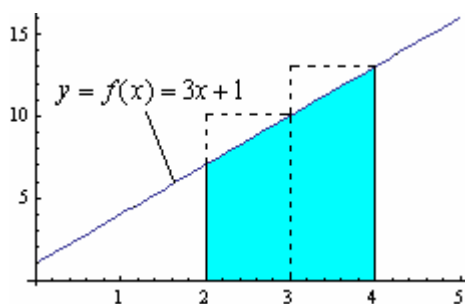
- ▶  $\frac{-1}{3}$
- ▶ 0
- ▶  $\frac{1}{3}$
- ▶  $\frac{2}{3}$

**Question No: 24 ( Marks: 1 ) - Please choose one**

$$\int_0^2 x dx = \underline{\hspace{2cm}}$$

- ▶ 2
- ▶ 0
- ▶ 2
- ▶ -2
- ▶  $\frac{x^2}{2}$
- ▶ 2

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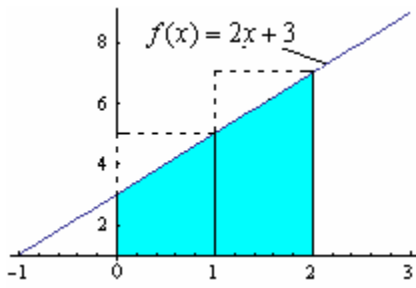


**Question No: 25 ( Marks: 1 ) - Please choose one**

Which of the following is approximate area of the shaded region by taking  $x_1^*$  and  $x_2^*$  as left endpoint of equal-length subintervals?

- ▶ 17
- ▶ 20
- ▶ 23
- ▶ 26

**Question No: 26 ( Marks: 1 ) - Please choose one**



Which of the following is approximate area of the shaded region by taking  $x_1^*$  and  $x_2^*$  as right endpoint of equal-length subintervals?

- ▶ 12
- ▶ 14

- ▶ 8
- ▶ 10

**Question No: 27 ( Marks: 1 ) - Please choose one**

What is the length of each sub-interval, if the interval  $[1,3]$  is divided into  $n$  sub-intervals of equal length?

- ▶  $\frac{1}{n}$
- ▶  $\frac{2}{n}$
- ▶  $\frac{3}{n}$
- ▶  $\frac{4}{n}$
- ▶

**Question No: 28 ( Marks: 1 ) - Please choose one**

Evaluate

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \text{-----}$$

- ▶ 4
- ▶ 2
- ▶ 1
- ▶  $\infty$

**Question No: 29 ( Marks: 1 ) - Please choose one**

$$\left\{ \frac{1}{2^n} \right\}_1^n$$

represents the sequence:

$$\frac{-1}{2}, \frac{-1}{4}, \frac{-1}{8}, \dots$$

- ▶



$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$$



$$0, 1, \frac{1}{2}, \frac{1}{4}, \dots$$



$$0, 1, 2, 3, \dots$$



**Question No: 30 ( Marks: 1 ) - Please choose one**

For a sequence  $\{a_n\}$  if the difference between successive terms  $a_{n+1} - a_n \leq 0$  then the sequence is known as:

- ▶ Increasing
- ▶ Decreasing
- ▶ Nondecreasing
- ▶ **Nonincreasing**

**Question No: 31 ( Marks: 1 ) - Please choose one**

For a sequence  $\{a_n\}$  if the ratio of successive terms  $\frac{a_{n+1}}{a_n} > 1$  then the sequence is known as:

- ▶ **Increasing**
- ▶ Decreasing
- ▶ Nondecreasing
- ▶ Nonincreasing

**Question No: 32 ( Marks: 1 ) - Please choose one**

If the partial sum of a series is finite then the series will/will be:

- ▶ Divergent
- ▶ **Convergent**
- ▶ Give no information

**Question No: 33 ( Marks: 1 ) - Please choose one**

If the geometric series  $a + ar + ar^2 + ar^3 + \dots + ar^{k-1} + \dots$  where  $(a \neq 0)$ ,  $|r| < 1$  then which of the following is true for the given series?

- ▶ **Converges**
- ▶ Diverges
- ▶ Gives no information

**Question No: 34 ( Marks: 1 ) - Please choose one**

If  $\rho = \lim_{k \rightarrow +\infty} \sqrt[k]{u_k}$  where  $\rho > 1$  then the series  $\sum u_k$  with positive terms will /will be.....?

- ▶ **Convergent**

- ▶ Divergent
- ▶ Give no information

**Question No: 35 ( Marks: 1 ) - Please choose one**

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$$

Which of the following is true for the series ?



- ▶ Arithmetic Series
- ▶ Geometric Series
- ▶ **Alternating Harmonic Series**
- ▶ Harmonic Series

**Question No: 36 ( Marks: 1 ) - Please choose one**

.....is the special case of Tylor's theorem.

▶ **Roll's Theorem**

- ▶ Picard's Method
- ▶ Integration
- ▶ Maclaurin's Theorem

**Question No: 37 ( Marks: 1 ) - Please choose one**

If  $f$  is integrable on a closed interval containing the four points  $a, b, c$  and  $d$  then

$$\int_a^d f(x) dx = \underline{\hspace{2cm}}$$

▶  $\int_a^b f(x) dx + \int_b^c f(x) dx + \int_c^d f(x) dx$

▶  $\int_a^b f(x) dx + \int_c^d f(x) dx$

▶  $\int_a^c f(x) dx + \int_b^d f(x) dx$

▶  $\int_a^d f(x) dx$

▶

**Question No: 38 ( Marks: 1 ) - Please choose one**

Suppose  $f$  and  $g$  are integrable functions on  $[a, b]$  and  $c$  is a constant, then

$$\int_a^b c [f(x) + g(x)] dx = \underline{\hspace{2cm}}$$

$$\int_a^b f(cx)dx + \int_a^b g(cx)dx$$



$$\int_a^b f(x) dx + \int_a^b g(x) dx$$



$$c \int_a^b f(x)dx + c \int_a^b g(x)dx$$



**Question No: 39 ( Marks: 1 ) - Please choose one**

What is the difference between the values of the

$$\int_a^b f(x)dx \text{ and } \int_a^b f(t)dt$$

integrals ?

- ▶ Differ by b-a
- ▶ Differ by a-b
- ▶ **No difference**
- ▶ Differ by b+a

**Question No: 40 ( Marks: 1 ) - Please choose one**

$$\int_{-1}^2 f(x) dx = 5 \quad \int_{-1}^2 g(x) dx = -3$$

If \_\_\_\_\_ and \_\_\_\_\_ then which of the following is value

$$\int_{-1}^2 [f(x) + 2g(x)] dx$$

of \_\_\_\_\_ ?

- ▶ **-1**
- ▶ -8
- ▶ 2
- ▶ 11

**Question No: 41 ( Marks: 2 )**

$$\frac{1}{1} + \frac{1}{8} + \frac{1}{27} + \dots + \frac{1}{1000}$$

Express the sum \_\_\_\_\_ in sigma notation.

$$\sum_{n=1}^{10} (1/n^3)$$

**Question No: 42 ( Marks: 2 )**

Only write down the Maclaurin series for  $e^x$

**Question No: 43 ( Marks: 2 )**

Evaluate the following integral:



$$\int_1^4 \sqrt{x} dx$$
$$\int_1^4 \sqrt{x} dx$$
$$= \int_1^4 \sqrt{x} \cdot 1 dx$$
$$= x\sqrt{x} + \int_1^4 1/\sqrt{x} \cdot 1 dx$$

**Question No: 44 ( Marks: 3 )**

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Evaluate the following sum:

$$\sum_{k=1}^6 (k^2 - 5)$$
$$= -4 - 1 + 4 + 11 + 20 + 31 = 61$$

**Question No: 45 ( Marks: 3 )**

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Find a definite integral indicating the area enclosed by the curves  $y = x^2$ ,  $x > 0$  and bounded on the sides by the lines  $y = 1$  and  $y = 4$ . But do not evaluate the integral.

**Question No: 46 ( Marks: 3 )**

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$$a_n = \left\{ \frac{3}{n^2} \right\}_{n=5}^{\infty}$$

Determine whether the following sequence is strictly monotone or not. If your answer is yes or no, then give reason .

Yes the sequence is strictly monotone because the denominator is increasing

**Question No: 47 ( Marks: 5 )**

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The region bounded by the  $y$ -axis, the graph of the equation  $x = y^{\frac{3}{2}}$  and the line  $y = 2$  is revolved about  $y$ -axis. Find the volume of the resulting solid.

**Question No: 48 ( Marks: 5 )**

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Compute the following sum:

$$\sum_{i=1}^n (4i^2 - i) = (4(1)^2 - 1) + (4(2)^2 - 2) + (4(3)^2 - 3) + (4(4)^2 - 4) \dots \dots \dots$$
$$= 3 + 14 + 33 + 60 \dots \dots \dots$$

**Question No: 49 ( Marks: 5 )**

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Use L'Hopital's rule to evaluate the limit

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{1 + \cos 2x}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} (1 - \sin x) = 0 \quad \lim_{x \rightarrow \frac{\pi}{2}} (1 + \cos 2x) = 0$$
$$= 0/0$$

So by L'Hopital's rule

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{1 + \cos 2x}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{d/dx(1 - \sin x)}{d/dx(1 + \cos 2x)}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{-\cos x}{-2 \sin 2x} = \frac{\cos \frac{\pi}{2}}{2 \sin \pi} = 0$$

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**Question No: 50 ( Marks: 10 )**

$$\sum_1^{\infty} \frac{2^n}{n(n+2)}$$

Use the Ratio test to determine whether the series converges or diverges.

$$p = \lim_{x \rightarrow \infty} \frac{u_{k+1}}{u_k} =$$

This paper is solved by our best knowledge. In the case of any error/correction/suggestion, please contact at [gulshanvu@yahoo.com](mailto:gulshanvu@yahoo.com), with reference to the concerned paper's number.