

FINAL TERM EXAMINATION
Fall 2009

Calculus & Analytical Geometry-I

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UPDATED VERSION

Exclusive thanks to Mahar Azahar (Lodhran)

Question No: 1 (Marks: 1) - Please choose one

Let $f(x)$ is a function such that as x approaches a real number a , either from left or right-hand-side, the function values increases or decreases unboundedly then

$$\lim_{x \rightarrow a} f(x)$$

- ▶ Exist
- ▶ Does not exist

Question No: 2 (Marks: 1) - Please choose one

$$\frac{d(\sec x)}{dx} =$$

- ▶ $(\sec x)(\tan x)$
- ▶ $(\sec x)(\tan x)$
- ▶ $(\operatorname{cosec} x)(\cot x)$
- ▶ $(\operatorname{cosec} x)(\tan x)$
- ▶

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Question No: 3 (Marks: 1) - Please choose one

Consider a function $h(x)$ and a constant c then

$$\frac{d}{dx}((c) \{h(x)\}) = \underline{\hspace{2cm}}$$

- ▶ 0
- ▶ $\frac{d}{dx}(h(x))$
- ▶ $\frac{d}{dx}(h(cx))$
- ▶ $c \frac{d}{dx}(h(x))$

Question No: 4 (Marks: 1) - Please choose one

If f is continuous function such that $\lim_{x \rightarrow -\infty} f(x) = +\infty$ and $\lim_{x \rightarrow +\infty} f(x) = +\infty$

then f has _____ on $(-\infty, +\infty)$

- ▶ maximum value but no minimum
- ▶ minimum value but no maximum
- ▶ both maximum and minimum value

Question No: 5 (Marks: 1) - Please choose one

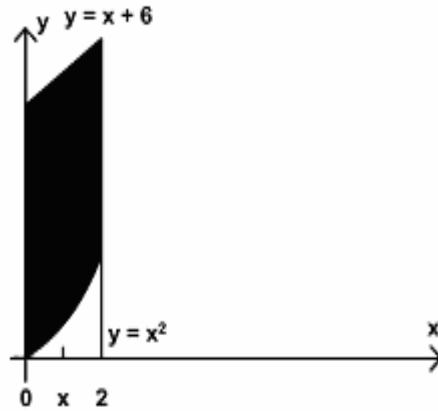
Sigma notation is represented by which of the following Greek letter?

- ▶ χ
- ▶ η
- ▶ Σ
- ▶ ψ

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Question No: 6 (Marks: 1) - Please choose one

In the following figure, the area enclosed is bounded below by :



- ▶ $y = x + 6$
- ▶ $y = x^2$
- ▶ $x = 2$
- ▶ $x = 0$

Question No: 7 (Marks: 1) - Please choose one

At what points the two curves: $y = x^2$ and $y = x + 6$ intersect ?

- ▶ $x = 0$ and $x = 2$
- ▶ $x = 0$ and $x = 3$
- ▶ $x = 2$ and $x = 3$
- ▶ $x = -2$ and $x = 3$

Question No: 8 (Marks: 1) - Please choose one

Let the solid generated by the region enclosed between

$$y = \sqrt{x} \quad ; \quad x = 1, x = 4$$

and the x-axis is revolved about the y-axis. Which of the following equation gives the volumes of a solid by cylindrical shells?

▶
$$V = \int_1^4 2\pi x \sqrt{x} dx$$

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$$V = \int_1^4 2x\sqrt{x} dx$$



$$V = \int_0^4 2x\sqrt{x} dx$$



$$V = \int_{-4}^4 2x\sqrt{x} dx$$



Question No: 9 (Marks: 1) - Please choose one

Let f is a smooth curve on the interval [a, b]. What is the arc length L of the curve f(x) defined over the interval [a, b]?

$$L = \lim_{\max \Delta x \rightarrow 0} \sum_{k=1}^n \sqrt{1 + (f'(x_k^*))}$$



$$L = \sum_{k=1}^n \sqrt{1 + (f'(x_k^*))} \Delta x_k$$



$$L = \lim_{\max \Delta x \rightarrow 0} \sum_{k=1}^n \sqrt{1 + (f'(x_k^*))^2} \Delta x_k$$



$$L = \sum_{k=1}^n \sqrt{1 + (f'(x_k^*))} \Delta x$$



Question No: 10 (Marks: 1) - Please choose one

For a graph to be symmetric about y-axis means, for each point (x,y) on the graph, the point ----- is also on the graph

▶ (x, -y)

▶ (-x, y)

- ▶ $(-x, -y)$

Question No: 11 (Marks: 1) - Please choose one

The graph $x = y^2$ is symmetric about -----axis

- ▶ X-axis
- ▶ **Y-axis**
- ▶ Origin

Question No: 12 (Marks: 1) - Please choose one

If a quantity y depends on another quantity x in such a way that each value of x determines exactly one value of y , we say that y is of x

- ▶ relation
- ▶ **function**
- ▶ not a function
- ▶ not a relation

Question No: 13 (Marks: 1) - Please choose one

$$\frac{(x^2 - 4)}{(x - 2)}$$

Domain of the function $y =$ is

- ▶ **$(-\infty, 2) \cup (2, +\infty)$**
- ▶ $(-\infty, 2)$
- ▶ $(-\infty, 0)$

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Question No: 14 (Marks: 1) - Please choose one

Tan(x) is continuous every where except at points

- ▶ **$\pm \frac{k\pi}{2} (k = 1, 3, 5, \dots)$**
- ▶ $\pm \frac{k\pi}{2} (k = 2, 4, 6, \dots)$
- ▶

$$\pm \frac{k\pi}{2} (k = 1, 2, 3, 4, 5, 6, \dots)$$



Question No: 15 (Marks: 1) - Please choose one

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \text{-----}$$

- ▶ -1
- ▶ 2
- ▶ 0
- 1

Question No: 16 (Marks: 1) - Please choose one

How the series $1 - 3 + 5 - 7 + 9 - 11$ can be expressed in sigma notation?

$$\sum_{k=0}^{k=5} (-1)^k (2k + 1)$$



$$\sum_{k=1}^{k=5} (-1)^k (2k + 1)$$



$$\sum_{k=1}^{k=5} (2k + 1)$$



$$\sum_{k=1}^{k=5} (2k + 1)$$



Question No: 17 (Marks: 1) - Please choose one

Let the region bounded by the curve $y = \sqrt[3]{x}$, the x-axis, and the line $x = 3$ is revolved about the y-axis to generate a solid. Which of the following equation gives the volume of a solid by cylindrical shells?

$$V = \int_0^3 x^{\frac{3}{2}} dx$$



$$V = 2\pi \int_0^3 \sqrt{x} dx$$



$$V = \int_0^3 2\pi x \sqrt[3]{x} dx$$



$$V = \int_0^3 x \sqrt[3]{x} dx$$



Question No: 18 (Marks: 1) - Please choose one

$$y = \frac{2\sqrt{2}}{3} x^{\frac{3}{2}} ; 0 \leq x \leq 2$$

Let _____ then which of the following is the length of the curve?

$$L = \int_0^2 \sqrt{\left[\frac{d}{dx} \left(\frac{2\sqrt{2}}{3} x^{\frac{3}{2}} \right) \right]^2} dx$$



$$L = \int \sqrt{1 + \left[\frac{d}{dx} \left(\frac{2\sqrt{2}}{3} x^{\frac{3}{2}} \right) \right]^2} dx$$



$$L = \int_0^2 \sqrt{1 + \left[\frac{d}{dx} \left(\frac{2\sqrt{2}}{3} x^{\frac{3}{2}} \right) \right]^2} dx$$



$$L = \int_0^2 \sqrt{1 + \left[\frac{d}{dx} \left(\frac{2\sqrt{2}}{3} x^{\frac{3}{2}} \right) \right]} dx$$



Question No: 19 (Marks: 1) - Please choose one

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$\frac{2}{3}$

is known as

- ▶ An even number
- ▶ Irrational Number
- ▶ A natural Number
- ▶ Rational Number

Question No: 20 (Marks: 1) - Please choose one

$$f'(x_n) = 0 \text{ for some } n$$

For a function f , let

Does the Newton's Method works for approximating the solution of $f(x) = 0$?

- ▶ Yes
- ▶ No

Question No: 21 (Marks: 1) - Please choose one

The Mean Value Theorem states that "Let function f be differentiable on (a,b) and continuous on $[a, b]$, then there exist at least one point c in (a,b) where"

▶ $f'(c) = \frac{f(b) - f(a)}{b - a}$

▶ $f(c) = \frac{f(b) - f(a)}{b - a}$

▶ $f(c) = \frac{f(a) - f(b)}{b - a}$

▶ $f'(c) = \frac{f(a) - f(b)}{b - a}$

▶

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Question No: 22 (Marks: 1) - Please choose one

$$\frac{d}{dx}[F(x)] = f(x)$$

If there is some function F such that

then any function of

the form $F(x) + C$ is ----- of $f(x)$

- ▶ Derivative
- ▶ Antiderivative
- ▶ Slope
- ▶ Maximum value

Question No: 23 (Marks: 1) - Please choose one

If f and g are continuous function on an interval $[a, b]$

and $f(x) \geq g(x)$ for $a \leq x \leq b$, then area is bounded by the lines parallel to:

- ▶ X -axis
- ▶ Y-axis
- ▶ Both X -axis and Y-axis

Question No: 24 (Marks: 1) - Please choose one

What is the sum of following series?

$$1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3$$

$$\frac{n(2n)(2n+1)}{6}$$



$$\frac{(n+1)(n+2)}{2}$$



$$\left[\frac{n(n+2)}{2} \right]^2$$



$$\left[\frac{n(n+1)}{2} \right]^2$$



Question No: 25 (Marks: 1) - Please choose one

$$\frac{5}{7} \times 1^2 + \frac{5}{7} \times 2^2 + \frac{5}{7} \times 3^2 + \frac{5}{7} \times 4^2 \dots + \frac{5}{7} \times n^2 = \underline{\hspace{2cm}}$$

$$\frac{5n(n+1)(2n+1)}{42}$$



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$$\frac{5n(n+1)}{14}$$



$$\frac{5n^2(n+1)^2}{14}$$



$$\frac{5(n+1)(2n+1)}{42}$$



Question No: 26 (Marks: 1) - Please choose one

$$\int_a^a f(x)dx = \underline{\hspace{2cm}}$$

If point a is in the domain of function f , then

▶ $f'(x)$

▶ $f(x)$

▶ 0

▶ 1

Question No: 27 (Marks: 1) - Please choose one

If $a_1 > a_2 > a_3 > \dots > a_n > \dots$, then a sequence $\{a_n\}$ is

▶ Increasing

▶ Nondecreasing

▶ Decreasing

▶ Nonincreasing

Question No: 28 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the difference between successive terms $a_{n+1} - a_n \leq 0$ then the sequence is known as:

▶ Increasing

▶ Decreasing

▶ Nondecreasing

▶ Nonincreasing

Question No: 29 (Marks: 1) - Please choose one

$$\frac{a_{n+1}}{a_n} < 1$$

For a sequence $\{a_n\}$ if the ratio of successive terms then the sequence is known as:

- ▶ Increasing
- ▶ **Decreasing**
- ▶ Nondecreasing
- ▶ Nonincreasing

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Question No: 30 (Marks: 1) - Please choose one

$$\frac{a_{n+1}}{a_n} \geq 1$$

For a sequence $\{a_n\}$ if the ratio of successive terms then the sequence is known as :

- ▶ Increasing
- ▶ Decreasing
- ▶ **Nondecreasing**
- ▶ Nonincreasing

Question No: 31 (Marks: 1) - Please choose one

$$a_n = \left\{ \frac{1}{n} \right\}_{n=1}^{\infty}$$

Which of the following option is true for the sequence ?

- ▶ Increasing
- ▶ **Decreasing**
- ▶ Nonincreasing
- ▶ Nondecreasing

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Question No: 32 (Marks: 1) - Please choose one

If the partial sum of a series is finite then the series will/will be:

- ▶ Divergent
- ▶ **Convergent**
- ▶ Give no information

Question No: 33 (Marks: 1) - Please choose one

If the geometric series $a + ar + ar^2 + ar^3 + \dots + ar^{k-1} + \dots$ where $(a \neq 0)$,

$|r| < 1$

then which of the following is true for the given series?

- ▶ Converges
- ▶ Diverges
- ▶ Gives no information

Question No: 34 (Marks: 1) - Please choose one

$\rho = \lim_{k \rightarrow +\infty} \frac{u_{k+1}}{u_k}$

If where $\rho > 1$ then the series $\sum u_k$ with positive terms will /will be.....?

- ▶ Convergent
- ▶ Divergent
- ▶ Give no information

Question No: 35 (Marks: 1) - Please choose one

$\rho = \lim_{k \rightarrow +\infty} \sqrt[k]{u_k}$

If where $\rho > 1$ then the series $\sum u_k$ with positive terms will /will be.....?

- ▶ Convergent
- ▶ Divergent
- ▶ Give no information

Question No: 36 (Marks: 1) - Please choose one

In alternating series test, which one of the following condition must be satisfied?

▶ $\lim_{k \rightarrow \infty} a_k = 1$

▶ $a_1 > a_2 > a_3 \dots > a_k > \dots$

▶ $a_1 \leq a_2 \leq a_3 \dots \leq a_k \leq \dots$

Question No: 37 (Marks: 1) - Please choose one

$\sum_{k=1}^{\infty} (-1)^n a_k$

A series of the form is called _____.



▶ Alternating series

▶ Geometric series

▶ Arithmetic series

▶ Harmonic series

Question No: 38 (Marks: 1) - Please choose one

Which of the following is the Maclaurin series for e^x ?

▶ $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^k}{k!} + \dots$

▶ $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^k}{k!} + \dots$

▶ $1 + x + \frac{x^3}{3!} + \dots + \frac{x^k}{k!} + \dots$

▶ $1 - x + \frac{x^3}{3!} - \dots - \frac{x^k}{k!} - \dots$

Question No: 39 (Marks: 1) - Please choose one

Which of the following is the work done W if an object moves in the positive direction along a coordinate line while subject to a force $F(x)$ in the direction of motion over an interval $[0,3]$?

▶ $W = \int_2^3 3x dx$

▶ $W = \int_0^3 3x dx$

▶ $W = \int_0^3 F(x) dx$

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$$W = \int_3^0 F(x)dx$$



Question No: 40 (Marks: 1) - Please choose one

Which of the following is the spring constant k if a spring whose natural length is $2m$ exerts a force of $3N$ when stretched $1m$ beyond its natural length?

- ▶ $3x$
- ▶ $3N/m$
- ▶ $2m$
- ▶ $3m/N$

Question No: 41 (Marks: 2)

Evaluate the following integral by substitution method.

$$\int x(2x^2 + 1)^{\frac{2}{3}} dx$$

Question No: 42 (Marks: 2)

Find the limits of the integral indicating the area bounded by the

curves $y = x^2$ and $y = x + 6$.

Sol,

Question No: 43 (Marks: 2)

What will be the amount of work done if an object moves $7m$ in the direction of a force of $70N$?

Question No: 44 (Marks: 3)

Evaluate the following integral:

$$\int \frac{5 - 6\sin^2 x}{\sin^2 x} dx$$

Question No: 45 (Marks: 3)

Find a definite integral indicating the area of the surface generated by revolving

the curve $y = \sqrt[3]{3x}$; $0 \leq y \leq 4$ about the x -axis. But do not evaluate the integral.

Question No: 46 (Marks: 3)

Find the spring constant K ; if a force of $10N$ is required to stretch a spring from its natural length of $4.8m$ to a length of $6.8m$?

Question No: 47 (Marks: 5)

$$\frac{d}{dx}[f(x)] = 12x^2 - 6x + 1$$

Let $\quad \quad \quad$. Find $f(x)$

Sol,

Question No: 48 (Marks: 5)

Use the cylindrical shell to find the volume of the solid generated when the region enclosed by the curve $y = x^3$, $x = 1$, $y = 0$ is revolved about the y -axis.

Question No: 49 (Marks: 5)

Determine whether the sequence $\{a_n\}$ converges or diverges; if it converges then find its limit;

$$a_n = \frac{3n^4 + 1}{4n^2 - 1}$$

where

Question No: 50 (Marks: 10)

Find the area of the region that is enclosed by the curves $y = x^2$ and $y = \sqrt{x}$

$$x = \frac{1}{4} \text{ and } x = 1$$

between $\quad \quad \quad$.

This paper is solved by our best knowledge. In the case of any error/correction/suggestion, please contact at gulshanvu@yahoo.com, with reference to the concerned paper's number.