

FINALTERM EXAMINATION 2009
(Session - 2)

Calculus & Analytical Geometry-I

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UPDATED VERSION

Exclusive thanks to Mahar Azahar (Lodhran)

Question No: 1 (Marks: 1) - Please choose one

According to Power-Rule of differentiation, if $f(x) = x^n$ where n is a real number, then

$$\frac{d}{dx}[x^n] =$$

- ▶ x^{n-1}
- ▶ **$n x^{n-1}$**
- ▶ $n x^{n+1}$
- ▶ $(n-1)x^{n+1}$

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Question No: 2 (Marks: 1) - Please choose one

If a function g is differentiable at a point x and a function f is differentiable at a point $g(x)$, then the _____ is differentiable at point x .

- ▶ **Composition (f o g)**
- ▶ Quotient (f / g)
- ▶ Product (f . g)
- ▶ Sum (f + g)

Question No: 3 (Marks: 1) - Please choose one

$$y = f(g(h(x)))$$

If

$$u = g(h(x))$$

$$v = h(x) \quad \frac{dy}{dx} = \underline{\hspace{2cm}}$$

then

▶ $\frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$

▶ $\frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$

▶

▶ $\frac{dv}{du} \cdot \frac{du}{dv} \cdot \frac{dy}{dx}$

▶

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Question No: 4 (Marks: 1) - Please choose one

If a function f is on a closed interval $[a,b]$, then f has both maximum and minimum value on $[a,b]$.

▶ Continuous

▶ Discontinuous

▶ None of these

Question No: 5 (Marks: 1) - Please choose one

$$\int_a^x \frac{t^2}{2} dt$$

The expression _____, represents a function of :

▶ t

▶ a

▶ Both x and a

▶ x

Question No: 6 (Marks: 1) - Please choose one

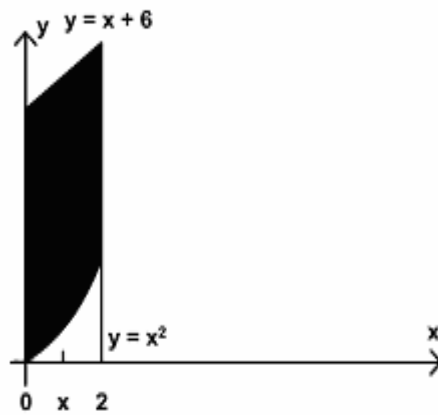
$$\int cf(x)dx = \underline{\hspace{2cm}}$$

if c is a constant

- ▶ 0
- ▶ c
- ▶ $\int f(cx)dx$
- ▶ $c \int f(x)dx$

Question No: 7 (Marks: 1) - Please choose one

In the following figure, the area enclosed is bounded below by :



- ▶ $y = x + 6$
- ▶ $y = x^2$
- ▶ $x = 2$
- ▶ $x = 0$

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Question No: 8 (Marks: 1) - Please choose one

At what points the two curves: $y = x^2$ and $y = x + 6$ intersect ?

- ▶ $x = 0$ and $x = 2$
- ▶ $x = 0$ and $x = 3$
- ▶ $x = 2$ and $x = 3$
- ▶ $x = -2$ and $x = 3$

Question No: 9 (Marks: 1) - Please choose one

What is the sum of following series?

$$1 + 2 + 3 + 4 + \dots + n$$

▶ $\frac{n+1}{2}$

▶ $\frac{(n+1)(n+2)}{2}$

▶ $\frac{n(n+2)}{2}$

▶ $\frac{n(n+1)}{2}$

▶

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Question No: 10 (Marks: 1) - Please choose one

If $b > 0$ then $\frac{d}{dx}[b^x] =$ _____

▶ 0

▶ xb^{x-1}

▶ $\ln b$

▶ $b^x \ln b$

Question No: 11 (Marks: 1) - Please choose one

Let S be a solid bounded by two parallel planes perpendicular to the x-axis at $x = a$ and $x = b$. If, for each x in $[a, b]$, the cross-section area of S perpendicular to the x-axis is $A(x)$, then what is the volume of the solid?

$$V = \int_a^b A(y) dx$$

▶

▶ $V = \int_a^b A(x) dx$

▶

$$V = \int_0^{A(x)} [b - a] dx$$



$$V = \int_0^{A(x)} [b + a] dx$$



Question No: 12 (Marks: 1) - Please choose one

Let the solid generated by the region enclosed between

$$y = \sqrt{x} \quad ; \quad x = 1, x = 4$$

and the x-axis is revolved about the y-axis. Which of the following equation gives the volumes of a solid by cylindrical shells?

$$V = \int_1^4 2\pi x \sqrt{x} dx$$



$$V = \int_1^4 2x \sqrt{x} dx$$



$$V = \int_0^4 2x \sqrt{x} dx$$



$$V = \int_{-4}^4 2x \sqrt{x} dx$$



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Question No: 13 (Marks: 1) - Please choose one

If slope m of a none vertical line is m = 1 then the angle of inclination of the line is

$$\frac{\pi}{4}$$



- ▶ $\frac{\pi}{2}$
- ▶ $\frac{\pi}{5}$

Question No: 14 (Marks: 1) - Please choose one

The PYTHAGORAS theorem describes the relationship between the sides of

- ▶ Right angle triangle
- ▶ Isoceleous triangle
- ▶ Equilateral triangle

Question No: 15 (Marks: 1) - Please choose one

If a quantity y depends on another quantity x in such a way that each value of x determines exactly one value of y , we say that y is of x

- ▶ relation
- ▶ function
- ▶ not a function
- ▶ not a relation

Question No: 16 (Marks: 1) - Please choose one

The graph of the equation $y = x^2 - 4x + 5$ will represent

- ▶ Parabola
- ▶ Straight line
- ▶ Two straight lines
- ▶ Ellipse



Question No: 17 (Marks: 1) - Please choose one

The $\lim_{x \rightarrow a} f(x) = \dots\dots$ where $f(x) = k$ (k is a constant)

- ▶ k+2
- ▶ k+1
- ▶ k

Question No: 18 (Marks: 1) - Please choose one

$$\int \frac{1}{x^2+2} dx$$

Consider the indefinite integral

Let $t = x^2 + 2$

Is the following substitution correct?

$$\int \frac{1}{x^2+2} dx = \int \frac{1}{t} dt$$

- ▶ Yes
- ▶ No

Question No: 19 (Marks: 1) - Please choose one

$$\log_b \frac{1}{t} = \underline{\hspace{2cm}}$$

- ▶ $\log_b t$
- ▶ $1 - \log_b t$
- ▶ $1 + \log_b t$
- ▶ $-\log_b t$



Question No: 20 (Marks: 1) - Please choose one

How the series $1 - 3 + 5 - 7 + 9 - 11$ can be expressed in sigma notation?

▶
$$\sum_{k=0}^{k=5} (-1)^k (2k+1)$$

$$\sum_{k=1}^{k=5} (-1)^k (2k + 1)$$



$$\sum_{k=1}^{k=5} (2k + 1)$$



$$\sum_{k=1}^{k=5} (2k + 1)$$



Question No: 21 (Marks: 1) - Please choose one

$$\sum_{k=1}^n f(x_k^*) \Delta x_k$$

The sum is known as:



- ▶ Riemann Sum
- ▶ General Sum
- ▶ Integral Sum
- ▶ Geometric Sum

Question No: 22 (Marks: 1) - Please choose one

$$\sum_{k=1}^n f(x_k^*) \Delta x_k$$

What does 'n' represent in the Riemann Sum ?

- ▶ No. of Circles
- ▶ No. of Subintervals
- ▶ No. of Loops
- ▶ No. of Squares

Question No: 23 (Marks: 1) - Please choose one

If w and v are continuous functions of y on an interval $[c, d]$

and $w(y) \geq v(y)$ for $c \leq y \leq d$, then area is bounded by the lines parallel to: :

- ▶ Y-axis
- ▶ X-axis
- ▶ Both X-axis and Y-axis

Question No: 24 (Marks: 1) - Please choose one

How the series $2(1) + 2(2) + 2(3) + 2(4) + 2(5)$ can be expressed in sigma notation?

▶ $\sum_{k=0}^5 2k^2$

▶ $\sum_{k=1}^5 2k^2$

▶ $\sum_{k=0}^5 2k$

▶ $\sum_{k=1}^5 2k$

Question No: 25 (Marks: 1) - Please choose one

$\sum_{k=1}^n \frac{k^3}{2} = \underline{\hspace{2cm}}$

▶ $\frac{n(n+1)}{4}$

▶ $\frac{[n(n+1)]^2}{8}$

▶ $\frac{n(n+1)(2n+1)}{12}$

▶ $\frac{(n+1)(2n+1)}{6}$

▶



Question No: 26 (Marks: 1) - Please choose one

If $a_1 < a_2 < a_3 < \dots < a_n < \dots$, then a sequence $\{a_n\}$ is.....

- ▶ Nondecreasing
- ▶ Decreasing

- ▶ Increasing
- ▶ Nonincreasing

Question No: 27 (Marks: 1) - Please choose one

If $a_1 \geq a_2 \geq a_3 \geq \dots \geq a_n \geq \dots$, then a sequence $\{a_n\}$ is

- ▶ Increasing
- ▶ Nondecreasing
- ▶ Decreasing
- ▶ Nonincreasing

Question No: 28 (Marks: 1) - Please choose one

If the difference between successive terms $a_{n+1} - a_n > 0$ then the sequence $\{a_n\}$ is known as:

- ▶ Increasing
- ▶ Decreasing
- ▶ Nondecreasing
- ▶ Nonincreasing

Question No: 29 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n} > 1$ then the sequence is known as:

- ▶ Increasing
- ▶ Decreasing
- ▶ Nondecreasing
- ▶ Nonincreasing



Question No: 30 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n} \geq 1$ then the sequence is known as :

- ▶ Increasing
- ▶ Decreasing
- ▶ Nondecreasing

- ▶ Nonincreasing

Question No: 31 (Marks: 1) - Please choose one

If $f(n) = a_n$ is the nth term of the sequence and $f'(n)$ is differentiable and $f'(n) > 0$ then the sequence will be:

- ▶ Increasing
- ▶ Decreasing
- ▶ Nondecreasing
- ▶ Nonincreasing

Question No: 32 (Marks: 1) - Please choose one

If the geometric series $a + ar + ar^2 + ar^3 + \dots + ar^{k-1} + \dots$ where $(a \neq 0)$, $|r| < 1$ then which of the following is true for the given series?

- ▶ Converges
- ▶ Diverges
- ▶ Gives no information

Question No: 33 (Marks: 1) - Please choose one

If the geometric series $a + ar + ar^2 + ar^3 + \dots + ar^{k-1} + \dots$ where $(a \neq 0)$, $|r| \geq 1$ then which of the following is true for the given series?

- ▶ Converges
- ▶ Diverges
- ▶ Gives no information

Question No: 34 (Marks: 1) - Please choose one

$$\sum_{k=1}^{\infty} u_k$$

$$\sum_{k=1}^{\infty} |u_k|$$

If the series $\sum_{k=1}^{\infty} u_k$ converges but the series $\sum_{k=1}^{\infty} |u_k|$ does not converge, then

$$\sum_{k=1}^{\infty} u_k$$

will _____

- ▶ Converge absolutely
- ▶ Diverge
- ▶ Converge conditionally



Question No: 35 (Marks: 1) - Please choose one

Let $\sum u_k$ be a series with nonzero terms and suppose that

$$\rho = \lim_{k \rightarrow \infty} \frac{|u_{k+1}|}{|u_k|} > 1$$

then which of the following is true?

The series $\sum |u_k|$ converges

The series $\sum |u_k|$ diverges

No conclusion can be drawn.

Question No: 36 (Marks: 1) - Please choose one

Suppose f and g are integrable functions on $[a, b]$ and c is a constant, then

$$\int_a^b c [f(x) + g(x)] dx = \underline{\hspace{2cm}}$$

$\int_a^b f(cx) dx + \int_a^b g(cx) dx$

$\int_a^b f(x) dx + \int_a^b g(x) dx$

$c \int_a^b f(x) dx + c \int_a^b g(x) dx$

0

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Question No: 37 (Marks: 1) - Please choose one

Which of the following is surface area S generated by revolving the curve $y = f(x)$ between $x = 0$ and $x = 2$ about the x -axis?

$$S = \int_0^2 2\pi f(x) \sqrt{1 + [f'(x)]^2} dx$$



$$S = \int_0^2 2\pi f(x) \sqrt{1 + [f'(x)]^2} dx$$



$$S = \int_0^2 2\pi f(x) \sqrt{1 + [f'(x)]} dx$$



$$S = \int_0^2 2\sqrt{1 + [f'(x)]} dx$$



Question No: 38 (Marks: 1) - Please choose one

Which of the following is area of the surface generated by revolving the curve

$$y = 4\sqrt{x} ; 1 \leq x \leq 4$$

about the x -axis?

$$\int_1^4 2\pi (4\sqrt{x}) \sqrt{1 + [(4\sqrt{x})]^2} dx$$



$$\int_1^4 2\pi (4\sqrt{x}) \sqrt{1 + [(4\sqrt{x})']^2} dx$$



$$\int_1^4 2\pi + \sqrt{1 + [(4\sqrt{x})']^2} dx$$



$$\int_1^4 2\pi (4\sqrt{x}) \sqrt{1 + [(4\sqrt{x})']^2} dx$$



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Question No: 39 (Marks: 1) - Please choose one

Which of the following is the work done W if an object moves in the positive direction along a coordinate line while subject to a force $F(x)$ in the direction of motion over an interval $[0,3]$?

$$W = \int_2^3 3x dx$$



$$W = \int_0^3 3x dx$$



$$W = \int_0^3 F(x) dx$$



$$W = \int_3^0 F(x) dx$$



Question No: 40 (Marks: 1) - Please choose one

$$\int_1^0 f(x) dx = 2 \quad \int_0^5 f(x) dx = 1$$

If $\int_1^5 f(x) dx$ and then which of the following is value of

$$\int_1^5 f(x) dx$$

- ▶ ?
- ▶ -3
- ▶ -1
- ▶ 1
- ▶ 3

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Question No: 41 (Marks: 2)

$$\frac{1}{2\sqrt{x}}$$

Derivative of a function is $\frac{1}{2\sqrt{x}}$. Find the original function.

Question No: 42 (Marks: 2)

If $\sum u_k$ is a series with positive terms and $\rho = \lim_{k \rightarrow +\infty} \sqrt[k]{u_k}$; then write the three cases for the series to be convergent, divergent or none.

Question No: 43 (Marks: 2)

Evaluate the following integral:

$$\int_1^3 \frac{1}{x^2} dx$$

Question No: 44 (Marks: 3)

Use the first fundamental theorem of calculus to evaluate the definite integral:

$$\int_0^2 f(x) dx \quad \text{where} \quad f(x) = \begin{cases} x^2 & ; 0 \leq x < 1 \\ x^3 & ; 1 \leq x \leq 2 \end{cases}$$

Question No: 45 (Marks: 3)

$$\sum_{k=2}^{\infty} (-1)^{k-1} \frac{2^{k-1}}{(k-1)!}$$

Show that the series converges absolutely.

Question No: 46 (Marks: 3)

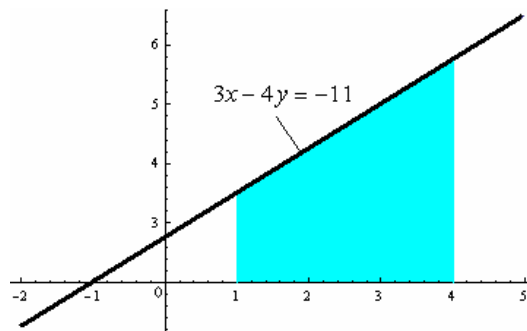
Express the following definite integral as limit of Riemann Sum. (Do not evaluate the integral)

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx$$

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Question No: 47 (Marks: 5)

Express area of the shaded region as a definite integral.



Question No: 48 (Marks: 5)

How much work is required to wind the chain onto the pulley if a 100-ft length of steel chain weighting 15 lb/ ft. is dangling from a pulley?

Question No: 49 (Marks: 5)

Evaluate the following integral:

$$\int_1^2 \frac{x^2 + \sqrt{x}}{x^2} dx$$

Question No: 50 (Marks: 10)

Use L'Hopital's Rule to evaluate

$$\lim_{x \rightarrow 0} (1 + \sin x)^{\cot x}$$

This paper is solved by our best knowledge. In the case of any error/correction/suggestion, please contact at gulshanvu@yahoo.com, with reference to the concerned paper's number.